PERGAMON

International Journal of Heat and Mass Transfer 43 (2000) 321–325

International Journal of
HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Technical Note

Transient vertical mixing by natural convection in a wide layer

Michael Epstein*, J.P. Burelbach

Fauske and Associates Inc., 16W070 West 83rd Street, Burr Ridge, IL 60521, USA

Received 28 January 1999; received in revised form 2 April 1999

1. Introduction

This work is motivated by flammable gas concerns in the headspaces of certain process waste storage tanks during episodic fuel–gas releases from the stored sludge/supernatant. The unsteady turbulent mixing of the light release fuel–gas with the overlying dense headspace air may be profitably regarded as a process analogous to Fickian (molecular) diffusion. A successful modeling approach to steady-state vertical mixing in spray and bubble columns is that of Baird and Rice [1]. Their formula for the turbulent diffusion (dispersion) coefficient E has been widely applied in modeling the upward turbulent transport of a lighter fluid through a heavier miscible fluid [2–6]. This formula is adopted here, and we write

$$E = l^2\left(\frac{g}{\rho}\frac{\partial\rho}{\partial z}\right)^{1/2} \tag{1}$$

where l is a characteristic mixing length, g is the gravitational constant and ρ is the density of the fluid mixture at vertical location z and time t. Eq. (1) is a modified but essentially equivalent version of Baird and Rice's original expression for E [6].

All previous experiments on density-gradient driven vertical mixing were carried out in high-aspect ratio tubes (or columns) and, as might be anticipated, the mixing length l in Eq. (1) was found to be a constant value proportional to the column diameter. In contrast, the geometry of the mixing region during a gas release event in a waste tank headspace is one of low aspect ratio. It is the purpose of this study to determine the mixing length l for mixing zones (layers) that are at least as broad as they are tall. This is achieved by assuming a priori that the mixing length l is linearly proportional to the instantaneous thickness $\delta(t)$ of the mixing layer, solving the transient mass balance (diffusion) equation for the case $l \sim \delta$ and comparing its solution with quantitative measurements of the mixing zone thickness-time history, obtained by injecting pure water into the bottom of a wide tank of sodium chloride salt solution (brine).

2. Asymptotic theory of vertical mixing in an unbounded gravitationally unstable brine/water layer

The problem we wish to solve is illustrated in Fig. 1, which shows a brine/water mixing layer of infinite extent in the horizontal direction. Initially the region $z > 0$ contains brine with a solute (salt) mass fraction Y_0. At time $t = 0$ pure water of density ρ_0 is injected uniformly at the bottom of the layer (at $z = 0$) with a known flux $\dot{m}''(0, t) = \rho_0 u_0$ where u_0 is the superficial injection velocity. The plane $z = 0$ is hereafter referred to as the porous surface or plate. We desire to develop expressions for predicting the time-development of the salt mass fraction profile $Y(z, t)$ and, more importantly, the instantaneous vertical extent $\delta(t)$ of the mixing layer.

For a dilute salt solution the mixture density ρ is a linear function of the salt concentration ρ_s:

---

* Corresponding author. Tel.: +1-630-887-5210; fax: +1-630-986-5481.

E-mail address: epstein@fauske.com (M. Epstein)

0017-9310/00/$ - see front matter
PII: S0017-9310(99)00136-2

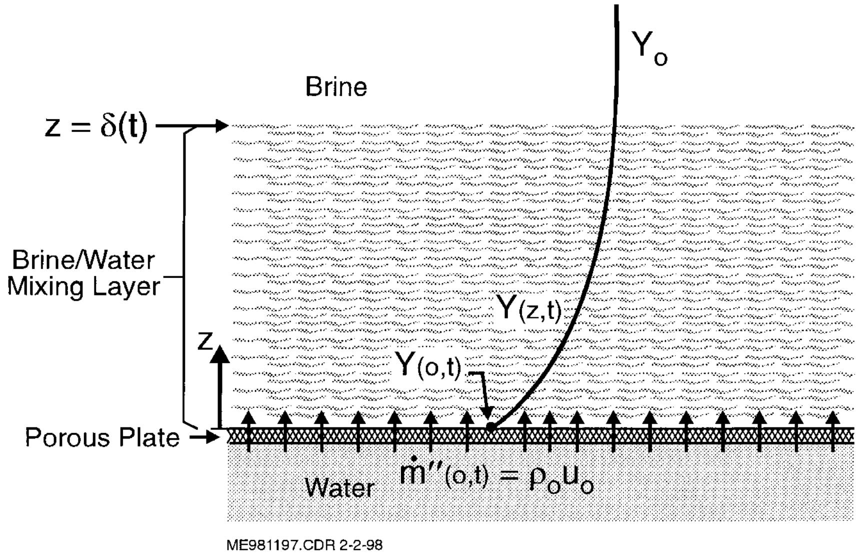

Fig. 1. Brine/water gravitational mixing model; illustrating solute (salt) concentration profile and water injection process.

$$\rho = \rho_0 + k\rho_s \tag{2}$$

where k is a constant (0.64 for NaCl solute). Substituting Eq. (2) into Eq. (1) and carrying out the indicated differentiation gives

$$E = l^2\left(\frac{gk}{\rho}\frac{\partial \rho_s}{\partial z}\right)^{1/2} \simeq l^2\left(gk\frac{\partial Y}{\partial z}\right)^{1/2} \tag{3}$$

The second equality in Eq. (3) is an approximation obtained by moving ρ inside the differential and invoking the definition $Y=\rho_s/\rho$. This step is permissible for dilute solutions. The solute mass balance (dispersion) equation is

$$\frac{\partial Y}{\partial t} = \frac{\partial}{\partial z}\left(E\frac{\partial Y}{\partial z}\right) = (gk)^{1/2}\frac{\partial}{\partial z}\left[l^2\left(\frac{\partial Y}{\partial z}\right)^{3/2}\right] \tag{4}$$

In writing Eq. (4) we have ignored terms related to molecular diffusion and convection due to water injection except at the porous surface (see below). For water injection velocities u_0 of practical interest, these terms are only important very early in the mixing transient and die away long before meaningful measurements of mixing zone thickness are possible in a laboratory experiment.

At the porous surface we apply the boundary condition

$$\dot{m}''Y(0,t) = \rho E\frac{\partial Y}{\partial z}, \quad \text{at } z=0 \tag{5}$$

Eq. (5) expresses the condition that the solute salt cannot pass through the porous surface. That is, the convective flux of salt due to the upward injection of fresh water is equal to the downward turbulent diffusion flux of salt to the surface. Again after a short induction period, the difference between $Y(0, t)$ in Eq. (5) and the initial salt concentration value Y_0 becomes small enough that Y_0 may replace $Y(0, t)$ in Eq. (5) with little loss in accuracy. Thus, Eq. (5) becomes [see also Eq. (3)]

$$u_0 Y_0 = (gk)^{1/2} l^2\left(\frac{\partial Y}{\partial z}\right)^{3/2}, \quad \text{at } z=0 \tag{6}$$

Finally, far above the porous surface (at $z=\infty$) we impose the conditions

$$Y(\infty,t) = Y_0, \quad \frac{\partial Y}{\partial z}(\infty,t) = 0 \tag{7}$$

The second condition in Eq. (7) implies zero flux at the upper boundary of the mixing layer.

Experience [3] with the class of diffusion problems represented by Eqs. (4), (6), and (7) suggests a similarity solution of the form

$$Y = Y_0 - At^p f(\eta) \tag{8}$$

$$\eta = \frac{Bz}{t^q} \tag{9}$$

where A, B, p and q are constant parameters whose values are yet to be determined. To close the problem the mixing length l is assumed to be constant over the instantaneous thickness δ of the mixing layer and proportional to it; so that

$$l = \beta\delta(t) \tag{10}$$

where β is as yet an unknown empirical constant. It follows from Eq. (9) that

$$\delta = \frac{\eta_\infty t^q}{B} \tag{11}$$

where η_∞ denotes the mixing layer thickness in 'similarity' space. Thus, from Eq. (10),

$$l = \frac{\beta\eta_\infty t^q}{B} \tag{12}$$

Substituting Eqs. (8) and (9) into Eqs. (4), (6), and (7) with l represented by Eq. (12) reduces the problem to the solution of the following ordinary-differential-equation boundary-value problem

$$3(-f')^{1/2}f'' + 3\eta f' + f = 0 \tag{13}$$

$$f'(0) = -1, \quad f(\infty) = f'(\infty) = 0 \tag{14}$$

where the superscript prime denotes derivatives with respect to η. The parameters A, B, p, and q are determined as byproducts of demanding that Eqs. (4) and (6) reduce to the dimensionless similarity forms given by Eqs. (13) and (14). The solute mass fraction profile given by Eq. (8) becomes

$$Y(z,t) = Y_0 - \frac{1}{\eta_\infty^2\beta^2}\left(\frac{u_0 Y_0}{gkt}\right)^{1/2} f(\eta) \tag{15}$$

and the thickness of the mixing zone given by Eq. (11) becomes

$$\delta(t) = \eta_\infty^3\beta^2(gku_0Y_0)^{1/2}t^{3/2} \tag{16}$$

Eq. (13) was solved numerically and the results indicate that $f(\eta)$ decreases monotonically from $f(0) = 1.4$ at $\eta = 0$ to a value of essentially zero at $\eta = 2.15$. On this basis the value $\eta_\infty = 2.15$ for the dimensionless mixing layer thickness seems appropriate.

3. Validity of asymptotic solution

It follows from Eq. (15) that the assumption $Y(0, t) \simeq Y_0$ in Eq. (5) is justified providing that the following condition is met:

$$\frac{(\eta_\infty\beta)^4 gkY_0 t}{f(0)^2 u_0} \gg 1 \tag{17}$$

This condition is satisfied for times greater than at most 1.0 s, which is short compared with the approximately 20 s time scale of the experiment mixing transient. Solute mass transport will be effectively turbulent diffusion controlled provided that the convective flux $u_0\ \partial Y/\partial z$ is small compared with the dispersion term $\partial/\partial z[E\ \partial Y/\partial z]$. By inserting the similarity variables into these terms we find that the criterion given by Eq. (17) also ensures negligible convective mass transport.

4. Experiment

Unsteady free convective mixing was investigated in an open rectangular tank having (inside) lateral dimensions of 0.892 × 0.740 m, giving a cross-sectional area of 0.66 m^2. The tank is 0.889 m high and is constructed of (transparent) acrylic plastic. A horizontal porous plate, fabricated from sintered stainless steel, divides the tank into two compartments. The lower compartment is 0.127 m high and forms a supply plenum for fresh water which is injected up through the porous plate during an experiment. The upper 'mixing' compartment is 0.762 m high and initially contains brine. The porous plate (supplied by Mott Corporation) is 1.57 mm thick, and its permeability is approximately $5 \times 10^{-13}\ \text{m}^2$ over the range of injection velocities (u_0) investigated.

Prior to running a series of mixing tests the tank is initially empty and the porous plate is clean and dry. Fresh water is then drawn from a storage tank via a valved pipe to fill the supply plenum and just cover the porous plate. With the supply plenum full, the upper mixing compartment is filled to the desired brine level. The effective diameter of the pores of the sintered porous plate is about 5.0 μm, which is small enough to prevent premature mixing of the water and brine across the thickness of the plate. The initial density of the brine is measured within ±1% either with a hydrometer or by weighing a liquid sample. The pressure in the fresh water storage tank is set to the desired value, a video camera (with a built-in 'stopwatch' timer) is turned on, and the water inlet valve is opened. Fresh water is then forced (injected) through the porous plate and into the brine compartment. Movement of the mixing zone, which is readily visible owing to refraction effects, is recorded by the video camera for later 'data reduction'.

Mixing zone visualization was enhanced by placing a grid of horizontal lines or wires of known spacing into the brine in the upper compartment. The best indication of the mixing front position was obtained by focusing on a given fixed horizontal line (wire) located a known distance above the porous plate. At some point in a mixing transient this line would represent a best estimate of the average mixing front position. During data reduction the videotape was played forward slowly (frame by frame if needed) to determine the time which best corresponded to each line (wire) in the visualization grid. These lines were typically positioned every one or two inches through the height of the upper 'mixing' compartment.

In addition to the grid a centimeter rule was attached to the side of the tank. The liquid level was measured before and after each test. Note that the rate at which the brine surface rose due to the injection of fresh water was negligible compared with the upward propagation of the brine/water mixing zone. The position of the brine layer surface was observed for a long elapsed time compared with the mixing zone growth time so that the injection velocity could be better determined from the brine surface position-time observations. Uncertainty in the injection velocity is estimated at ±6%.

Uncertainty in the position of the brine/water mixing front tended to increase with increasing front displacement. Early in a mixing transient, say while the mixing zone was within 10 cm above the porous plate, the mixing front was relatively 'flat'. However, as the front traveled farther into the brine region the mixing front became increasingly turbulent and non-uniform. Also, for deeper brine layers (or lower injection velocities) the increasing dilution of the mixing zone made the mixing front more difficult to observe. The time at which the mixing front crossed a given marker line was typically observable within ±0.5 s at 95% confidence during a 20 s transient. Late in a run this uncertainty could be as much as ±1 s. The scatter in

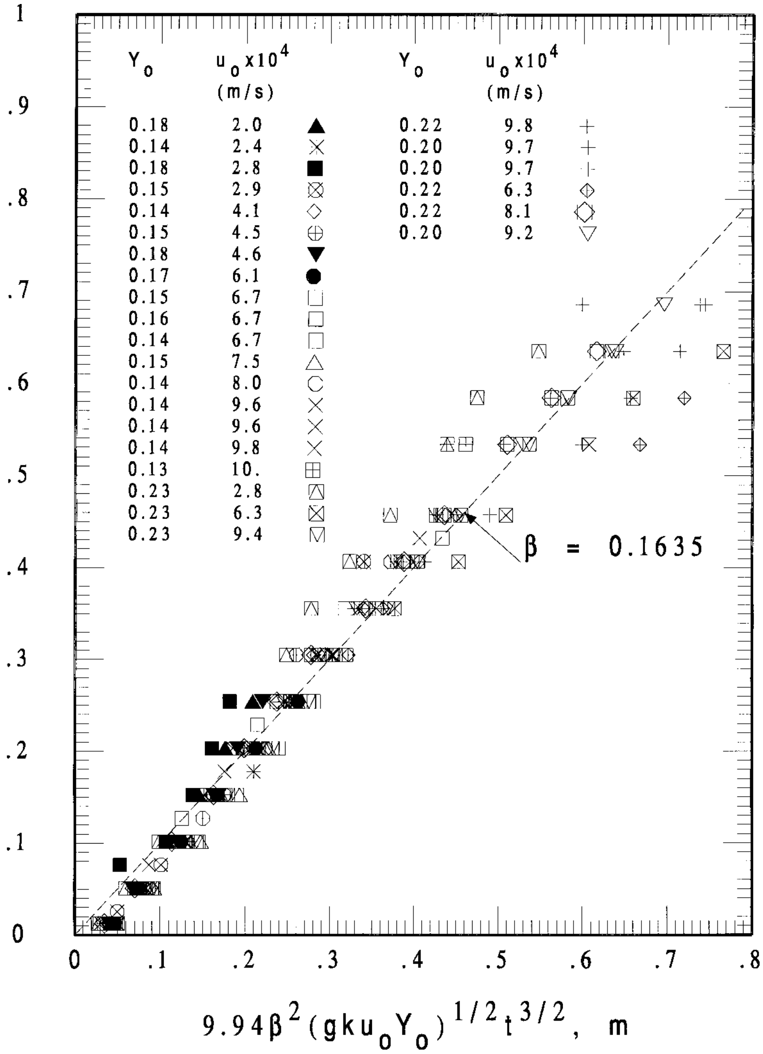

Fig. 2. Correlation of concentration front vs time data. Comparison with theory using characteristic mixing length $l = \beta\delta$.

the reported data (see Fig. 2) provides a good measure of the overall experimental uncertainty, which is estimated to be ±20%.

5. Experimental results and concluding remarks

The measured mixing zone thickness-time histories are plotted in Fig. 2 in a form suggested by Eq. (16) with $\eta_\infty = 2.15$. The dashed line in the figure is a least-squares fit of all the data from which we infer a mixing-length-thickness proportionality constant coefficient $\beta = 0.164$ [see Eq. (10)]. The correlation displayed in the figure is considered quite good as the data scatter is well within the range that one usually finds in gravitational mixing problems.

In previous studies of density-gradient driven mixing, high aspect ratio columns or tubes were employed and the Baird–Rice [1] mixing length l was found to be a constant proportional to the column diameter. In contrast, the brine/water data obtained in the present study of buoyant mixing in layers of considerable lateral extent indicate that the appropriate Baird–Rice mixing length is proportional to the instantaneous thickness of the mixing zone. In closing we note that there exists a simple analogy between mixing in the brine/water system and mixing in a heavy gas/light gas system so that the theory presented here may be readily converted to equations that allow useful predictions of the light release fuel-gas concentration profile during a gas release event.

References

[1] M.H.I. Baird, R.G. Rice, Axial dispersion in large unbaffled columns, Chem. Eng. J. 9 (1975) 171–174.

[2] M.H.I. Baird, N.V. Ramo Rao, Axial mixing in a reciprocating plate column in presence of very small density gradients, AIChE J. 37 (1991) 1019–1026.

[3] M.H.I. Baird, K. Aravamudan, N.V. Ramo Rao, J. Chadam, A.P. Peirce, Unsteady axial mixing by natural convection in a vertical column, AIChE J. 38 (1992) 1825–1834.

[4] M. Epstein, Buoyancy-driven exchange flow through small openings in horizontal partitions, J. Heat Transfer 110 (1988) 885–893.

[5] G.C. Gardner, Motion of miscible and immiscible fluids in closed horizontal and vertical ducts, Int. J. Multiphase Flow 3 (1977) 305–318.

[6] T.L. Holmes, A.E. Karr, M.H.I. Baird, Effect of unfavorable continuous phase density gradient on axial mixing, AIChE J. 37 (1991) 360–366.